An Exactly Soluble Model Relating Undergraduate Performance Indicators

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INTRODUCTION

When one wishes to introduce performance indicators in undergraduate programs, many difficulties appear relating independent and dependent variables. Moreover, the type of data collected, if aggregate, which considers number of enrollments but not specific students, or by coorts, which identifies the trajectory of every different student, lead to different values and interpretations.

In terms of performance indicators, the way of calculating annual attrition, persistence to degree rates, average total enrollment by new enrollment, average graduation time, and others, is strongly dependent on intrinsic academic variables such as annual average retention rates, different retention rates between the first year and the following years and rates of student failure to proceed for the next year of the course.

In order to help clarifying some of those relations, it is proposed, in this work, an exactly soluble model for the trajectory of students through an academic program towards a graduate degree, in a four year course, with a constant number of new enrollments in the course, throughout the years, a persistent rate for the first to second year transition (u), a different persistence rates for the other transitions (t), persistence rate when failure occurs beyond the first academic year (t’) and a probability of permanence in the same academic level for two successive years due to an academic failure (f).

It is shown, how differences in the dependent variables vary with the independent variables, several real data are examined. Finally, the elasticity of enrollment and the probability of students obtaining the degree with the persistence rates are calculated.

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ATTRITION RATES

Attrition in an educational institution is defined in two different points of view:

1 - The average annual attrition rate, which measures the percentage of students that reenrolled for the next academic year and the total number of students that could, in principle, reenroll.

In this case, the average annual attrition rate (AAAR) should be given by:

$$AAAR = 1 - [M(n)-N(n)]/[M(n-1)-G(n-1)], \qquad (1)$$

where M(n) is the total enrollment in year n, N(n) is the number of new students in year n and G(n-1) is the number of students the graduated in year n-1.

This calculation is somewhat different from the one used in several works, where the same function is defined as:

$$AAAR = [M(n-1) – G(n-1) – M(n) + N(n)]/M(n-1), \qquad (2)$$

which differs from the previous definition (1) by the G term in the denominator, leading to an attrition rate around 10% smaller as compared to the more exact expression. (There is no possibility of M(n-1) students reenroll since G(n-1) graduated and therefore are naturally expected to leave the institution).

2-The attrition as the complement of the persistence towards graduation, i.e., the percentage of students that begun the studies but never obtained the degree. It depends, not only on the attrition rates, but also on how those rates are divided throughout the course (for instance, higher for first to second year) and academic failure rates.

THE MODEL

In a constant number of first year new enrollments, all dependent variables will be proportional to this number. Therefore, everything can be calculated as probabilities per new incoming student.

As such, the number of total enrollments is calculated as the sum of all students in the first years study plans, plus those in the second year, those in the third and, finally, those in the fourth. No matter when they started studying.

For those in the first year we consider the new student, the one who failed once to be promoted to the second year, those who failed twice, and so on. Therefore we have:

First year enrollments (YEAR1) = $1+fu+f^2ut'+f^3ut'^2+\ldots$,

or,

YEAR1 = 1+uf/(1-ft), summing up the infinite geometrical series.

Similarly, for students in second academic year:

$$\text{YEAR1} = (1-f)u+2(1-f)fut'+3(1-f)f^2ut'^2+\ldots= (1-f)u \sum_{n=0}^{\infty} \binom{n-1}{1} (ft')^n \qquad (3)$$

where (1-f) are the promoting chances, and the natural numbers 1, 2, 3, etc, indicate the multiplicity of student trajectories leading to the same academic situation.

From the general Taylor expansion:

$$1/(1-x)^p = \sum_{n=0}^{\infty} \binom{n+p-1}{p-1} x^n , \qquad (4)$$

It is easy to write the enrollment for the second year as:

$$\text{YEAR2}=(1-f)u[1+2ft'+3f^2t'^2+\ldots]=(1-f)u(\partial / \partial (ft'))[1+ft'+(ft')^2+\ldots]=(1-f)u/(1-ft')^2$$

The same calculation can be applied to the next academic years, leading to enrollment in year N:

$$\text{YEARN} = (1-f)^{N-1}ut^{N-2}/(1-ft')^N, \text{ for } N>=2 \qquad (5)$$

The total enrollment in a course o N years (in Brazil, 4 for Administration, 5 for Engineering, 6 for Medicine, etc.) total enrollment (TE) per first year enrollment will be:

$$\text{TE} = \sum_{k=1}^{N} \text{YEARk}. \qquad (6)$$

The graduation rates (or persistence to degree) is simply the product of those reaching the fourth year of the curriculum times the probability of satisfactory academic performance, i.e.;

Graduation rates (GR) = $(1\text{-}f)^N ut^{N\text{-}1}/(1\text{-}ft)^N$. (7)

As another interesting parameter, the average graduation time (AGT) can also be calculated, and it is easy to shown to it is given by:

AGT = N/(1-ft’). (8)

EXAMPLES

1 - Applying the above formulas to the Brazilian higher education system, where the total enrollment per new entrant is 2,5, the graduation rate is 52%, and the average annual attrition is 22%, it is easy to verify that, for the data to be compatible, the average student failure should be around 10% and that attrition for the first year should be about three times that for the following years.

Moreover, assuming t = t’, it is possible to verify that the data for private and public higher education institutions are consistent in the model with the following assumptions:

Private system:

Data	Variable
AAAR = 27%	u = 0,6
PT = 42%	t = 0,9
TE = 2,5	f = 0,1

Public system:

Data	Variable
AAAR = 12%	u = 0,9
PT = 60%	t = 0,85
TE = 3,5	f = 0,1

2 - Elasticity of enrollment: The elasticity can be defined as the ratio of the relative variation of enrollment caused by the variation of the attrition rate:

$$\text{Elasticity} = (\partial E / \partial t) t / E \qquad (11)$$

For an average attrition rate of 30% and an average academic failure of 20%, the elasticity of enrollment in terms of attrition is –0,64, i.e., if attrition is reduced by 10%, total enrollment should increase 6,4%.

3 - To verify the importance of the first to second year retention rates, we analyze four hypothetical types of higher education institutions:

a- Institution A with a retention ratio of 70% from the first to second academic year and 90% thereafter and an average 10% index of academic failure for students;

b- Institution B, exactly the same as above but with zero average failures (all students succeed);

c- Institution C with the same persistence rates for every academic transition, of 82,2%, and a 10% average academic failure (such that it has the same average persistence rates as institution A);

d- Institution D with no academic failures and a persistence rate of 81,4% all over the course, i.e., for all academic transitions (such that it has the same average persistence rates as institution B).

Table 1 illustrates the main academic performances of the four institutions. As table 1 shows, for two institutions with the same average annual persistence rates, if enrollment rates are somewhat smaller for institutions with smaller persistence rates for first to second year transition than for those with a uniform distribution, the opposite occurs for the graduation rates, where they show a better performance. Moreover, institutions with lower failure rates have fewer students per new enrollment, but perform better in terms of graduating their students.

Of course, no one expects a zero failure rate, which should be taken as a limit, but the result indicates that increasing the academic success of students may endanger the financial equilibrium of a higher education institution, and if this is considered, as it should, as a noble proposal, efforts must be made to increase the retention rates in order to compensate such reduction. It can be shown that, for t'~ 0,8t, the effect of increasing enrollments through academic failures ceases to be important.

	Institution A	Institution B	Institution C	Institution D
First-second year persistence rates	70%	70%	82,2%	81,4%
Other transitions persistence rates	90%	90%	82,2%	81,4%
Average academic failures	10%	0%	10%	0%
Average annual persistence rate	82,2%	81,4%	82,2%	81,4%
Total enrollments per new student	3,12	2,90	3,25	3,02
Graduation rates	54,2%	56,7%	51,4%	54,0%

4 - Solving exactly, for a 3 year course, for u and t, knowing AAAR and PT. This assumes t = t', and f = 0.

Simple manipulation of the formulas leads to:

u = (1-AAAR-GR)/AAAR

and,

t = GRxAAAR/2x(1-AAAR-GR)

Since u and t should be ≤ 1, it can be shown that the evasion and graduation rates variables must obey some inequalities:

1-AAAR-GR$\leq$AAAR$\leq$(1-AAAR-GR)/GR.

5- Analysis of data for a real course in Brazil: In 2005, the course of Business Administration registered the following data:

TE = 2,47

GR = 0,39

AAAR = 0,30

Form these data it was possible to identify the following intrinsic parameters for the attrition process:

u = 0,61

t = 0,80

f ~ 0.

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